## **Analytic signal - 3D total gradient**

The concept of analytic signal applied to magnetic anomalies was developed in two dimensions by Nabighian (1972) based on a concept initially proposed by the French Ville in 1948. In two dimensions, the complex analytic signal of the magnetic signal can be expressed as (Thurston and Smith 1997):

$$A\left(x,y\right)=\left|A\left(x,y\right)\right|.exp⁡(jφ)$$

with

$$\left|A(x,y)\right|=\sqrt{\left(\frac{∂M}{∂x}\right)^{2}+\left(\frac{∂M}{∂y}\right)^{2}}$$

and

$$φ=tan^{-1}\left(\frac{∂M}{∂x}\right)$$

|A| is the 2D analytic signal amplitude, ϕ the local phase. A common theme of the normalized derivatives is the concept of mapping angles (or functions of angles) derived from the gradients of the magnetic intensity.

Using 3D dimensional grid, the amplitude of the analytic signal Aof M(x,y,z) is calculated by taking the square root of the sum of the squares of each of the directional first derivatives of the magnetic field.

$$\left|A(x,y)\right|=\sqrt{\left(\frac{∂M}{∂x}\right)^{2}+\left(\frac{∂M}{∂y}\right)^{2}+\left(\frac{∂M}{∂z}\right)^{2}}$$

The resulting shape of the analytic signal is expected to be centered above the magnetic body (Figs. .8, .9). This has the effect of transforming the shape of the magnetic anomaly from any magnetic inclination to one positive body-centered anomaly at least in 2D (Nabighian 1972). Analytic signal has been utilized widely for mapping of structures and for determining the depth of sources (Roest et al. 1992; Florio et al. 2006; Pilkington et al. 2000).

The main advantage of the total gradient over the maximum horizontal gradient is its lack of dependence on dip and magnetization direction, at least in 2D. When interpreting the analytic signal it is assumed that the causative sources are simple near-vertical or step-like geological structures (Roest et al. 1992; Roest and Pilkington 1993). Therefore, the 2D analytic signal has significant advantages over the simple derivatives and this application was utilized to map changes in basement structure, fabric and trends. Synthetic modelling has proved that the maxima of the analytic signal are located over the edge of anomalous sources (Nabighian 1974, 1984, Roest et al. 1992). This simplification of the potential field, however, results in the compromise whereby during computation the sign of the original gravity and magnetic field is lost. Therefore, it cannot be determined whether the analytic signal anomaly represent positive or negative density or magnetic susceptibility contrast compared to its surroundings.

However, it should be reminded that the total gradient expressed in 3D is not so independent of the direction of magnetization (Xiong 2006), nor does it represent the envelope of both the vertical and horizontal derivatives over all possible directions of the Earth’s field and source magnetization. Thus, despite its popularity, the 3D total gradient is not the correct amplitude of the real analytic signal in 3D. In a recent review, it appears that what is commonly called the analytic signal should be rather called "total gradient" in 3D (Nabighian et al. 2005).

In the 3D case, some factors produce offsets that are more difficult to predict than in the 2D case. Such factors primarily include the interference from neighbouring magnetic bodies, or from varying magnetization directions, terrain effects, 3D corners on body edges, or irregular boundary. Roest et al. (1992) have demonstrated that this is true for any 2D anomaly and suggested their generalization to 3D. However, the data should be interpreted with care in the 3D case. The properties of the analytic signal are strictly valid for isolated 2D bodies and one should use caution in extrapolating the conclusion of the 2D case to the 3D case. For 3D case, the shape and the absolute value of the analytic signal are also dependent of the directions of magnetization and of the Earth's field. The amount of offset is primarily determined by the depth to the top edge of the boundary below the observation level and by the dip of the boundary and by the directions of magnetization and of the Earth’s field. In the most common 3D cases, the locations of the total gradient are always offset from edges of the causative bodies. The best results are obtained for vertical and relatively shallow thick dikes (Xiong 2006). For the 2D finite dipping step model, the locations of the maximas vary with both burial depth and dipping angle. All these factors become important as the size of study area becomes smaller because the small amounts of offset they cause become more significant. For local scale magnetic surveys, mutual interference from different bodies is the greatest limiting factor of the analytic signal method and this can render the method virtually useless. For large-scale magnetic surveys like the NB-07 or for shallower depths of causative bodies, the analytic signal method remains nevertheless a useful and fast way of delineating magnetic boundaries in the subsurface (Fig. 5.8).

The technique therefore requires interpretation in conjunction with other geophysical and geological information to maximize its potential. After calibration with known structures or other derived potential field products the analytic signal can be interpreted geologically with better confidence.