Regional similarities in the distributions of well yield from crystalline rocks in Fennoscandia

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Well yields from Precambrian and Palaeozoic bedrock in Norway, Sweden and Finland exhibit very similar and approximately log-normal distributions: all three data sets exhibit a median yield of 600–700 L hr⁻¹, despite the differences in climate and lithology. This similarity is tentatively reflected on a larger geographical scale by a meta-analysis of the international data sets on crystalline rock aquifers from other recently glaciated areas (i.e., without a thick regolith of weathered rock). An heuristic treatment of the Fennoscandian data sets suggests that this median yield is consistent with the following bulk properties of shallow (to c. 70–80 m depth) crystalline bedrock: transmissivity of 0.56 ± 0.30 m² d⁻¹ (6.4 ± 3.4 x 10⁻⁶ m² s⁻¹) and hydraulic conductivity of around 1.1 (± 0.6) x 10⁻⁷ m s⁻¹.

Objective

The behaviour of fractured, crystalline rock aquifers is often regarded as being extremely complex, requiring a consideration of discontinuous flow systems, anisotropy and heterogeneity. Emphasis is often placed on the importance of local tectonic, topographic and weathering controls for the development of transmissivity. Without wishing to detract from the validity of these ‘complex’ approaches, this paper considers well yield data at regional and national scales, to demonstrate that large data sets from such aquifers exhibit considerable similarity in statistical distribution, despite apparent differences in lithology and climate. This paper discusses the implications of this finding and heuristically attempts to deduce what this may imply about the distributions of transmissivity in crystalline rock aquifers.

The authors specifically consider crystalline bedrock aquifers which are dominated by tectonically controlled fracture systems: i.e., crystalline bedrock in recently glaciated terrain, where geological time has not been sufficient to permit the development of a significant overlying weathered regolith or saprolite. Such a regolith is characteristic of crystalline bedrock in temperate/tropical climates (e.g., Africa and India and also parts of southern and central Europe) and its properties tend towards a conventional ‘porous medium’ aquifer. In regions where it overlies a true fractured crystalline bedrock aquifer, the presence of regolith may be expected to modify the behaviour and prop-
erties of the bedrock aquifer system. Nevertheless, towards the conclusion of this paper, the findings from recently glaciated Fennoscandian and North American crystalline rock aquifers are tentatively compared to tropical crystalline rock terrains to ascertain whether any of the conclusions from the former may be transferable to the latter.

Introduction

It is widely accepted that wells in poorly weathered, fractured, crystalline bedrock aquifers have groundwater yields that are impossible to predict deterministically. Many researchers (Rohr-Torp 1994, Henriksen 1995, 2003, 2006, Sander 1997, 2007, Morland 1997, Henriksen and Braathen 2006) have attempted to correlate well yields in such terrains with factors such as lithology, topography, proximity to lineaments and postglacial isostatic rebound and have found some degree of tentative correlation. Often, however, the degree of correlation has been less than impressive—to cite a few of these articles:

“Results from correlation studies between mapped lineaments and well capacities have been ambiguous and demonstrate that lineaments alone cannot be used for water well siting.” (Sander 1997).

“The results of this exploratory data analysis do not indicate any clear trends in well yield with regional gradients.... a simple correlation based on the relationships between two variables would not be sufficient to describe the observed regional variations in well yields.” (Henriksen 2003).

“The testing of the two hypotheses” [i.e., regarding correlation between well yield and position relative to a lineament] “does not give a clear and unequivocal answer in support of the two assumptions about groundwater flow in the study area.” (Henriksen and Braathen 2006).

“The hunt for major lineaments and lineament intersections as targets can be discouraging...” (Sander 2007).

It thus remains an important observation that, while one can subdivide a set of water wells on the basis of an attribute such as lithology or tectonic setting, the systematic differences in groundwater yield between subsets remain rather modest and the variation within each subset is enormous. Because of this, several authors have argued that it is more meaningful to make probabilistic assessments of well yield, on the basis of nonparametric statistics, rather than deterministic predictions (Gustafson 2002, Banks et al. 2005). Banks and Robins (2002) have likened well drilling in such terrain to a strategic game with a high element of chance, such as poker, rather than to a deterministic battle of wits, such as chess. Intriguingly, Braester and Barak (1991) explicitly considered drilling in fractured rock terrain as a two-player, zero-sum game (a mathematical term denoting a game where the gains of one player balance the losses of another), where the well-driller pits his luck against Mother Nature.

If one wishes to pursue correlations between well yields and geological, tectonic or lithological factors, it is beyond doubt that large, quality-controlled data sets and appropriate statistical techniques and tools are essential. However, it can be argued that the search for such correlations has overshadowed the significance of the far more interesting observation that the frequency distributions of water well yields in crystalline bedrock aquifers from nations such as Norway, Sweden and Finland are remarkably similar, irrespective of varying climate, tectonic setting, Quaternary history and lithology. Indeed, Krásný and Sharp (2007) go further and state that transmissivity distributions are rather similar in near-surface crystalline rock aquifers in many other parts of the world, including Korea, Poland, the Czech Republic and Ghana.

This paper will systematically compare, for the first time, the overall statistical distribution of well yields within the Fennoscandian nations (Norway, Sweden, and Finland) (Figure 1).

Figure 1. Map of Norway, Finland and Sweden. Showing Central Finland, Skåne (Scania) and the Tornqvist Zone (TZ).

The distribution of well yields in databases

Most nations attempt to collect details of the location, penetrated geology, construction and production data of water wells in databases, with varying degrees of success. A good well data-
A database is, naturally, a prerequisite for any statistical analysis of well yields. Such a database should ideally be:

1. Large: in order to provide a good degree of confidence in calculated mean, median or percentile yields.
2. Representative: there is a clear danger that ‘failed’ (i.e., poorly yielding) wells will tend to be under-reported to national databases, relative to successful wells that are eventually taken into production.
3. Reliable: yield data, often of variable quality, should ideally be quality-filtered to secure reliable statistical analysis (e.g., Morland 1997, Henriksen 2008).

In Scandinavia, and especially in Norway and Sweden, these criteria are fulfilled (albeit imperfectly) to a greater degree than in many other countries. The Scandinavian nations generally have a culture that prizes the curation of information. The databases hosted by the national Geological Surveys of Norway (NGU) and Sweden (SGU) contain tens of thousands of records of crystalline rock water wells. (The Geological Survey of Finland (GTK) has a nationwide database of a few thousand drilled wells with comprehensive water quality information but, unfortunately, sparse well yield data.) Furthermore, because the nations are rather small and the numbers of operational drilling companies limited, the Surveys have often been able to establish rather good relationships with the most important drillers to ensure representative reporting. Of course, this does not rule out the possibility of ‘failed’/poorly yielding wells being underrepresented, especially in historic data, but the fact that most hard rock wells are drilled for individual households (which may only have a requirement of <100 L hr⁻¹ water) means that a well has to be very poorly yielding indeed to be deemed a ‘failure’. One must also accept the possibility that unscrupulous drillers (who are surely in a small minority!) may ‘inflate’ well yields, with confidence that a householder will seldom independently put these to the test. Nevertheless, one may argue that the Swedish and Norwegian databases are amongst the best and most representative databases of crystalline bedrock wells available today. In Finland, as in Norway and Sweden, crystalline bedrock aquifers are a major source of groundwater supply in rural areas (Mäkelä 1993). While no national Finnish database has been constructed and statistically analysed in the same way that Norwegian and Swedish data have been, representative regional databases have been examined in parts of Finland (Rönkä 1993).

Despite the different lithologies, climates and tectonic histories of Sweden, Finland and Norway, it is remarkable that the data distributions for well yield from crystalline bedrock databases in the three nations are very similar (Figure 2), all with a median well yield of 600–700 L hr⁻¹. Note that Figure 2 (and Figure 4) has a probability scale on the y-axis and a logarithmic scale on the x-axis. On such log-probability plots, a log-normal distribution yields a straight line.

Morland (1997) was responsible for the first comprehensive statistical analysis of the Norwegian crystalline bedrock well database in recent years. After rigorous quality control, he ended up with a set of 12,757 records. He subdivided the records on the basis of lithology and found that, while a few specific lithologies (e.g., the Permian lavas of the Oslo Rift, some sandstone and marble lithologies) (Table 1) exhibited significantly higher yields, the bulk of the lithologies had median well yields of several hundred L hr⁻¹. He found, moreover, that within each lithology there was a huge range in well yield, from effectively ‘dry’ (nominally 10 L hr⁻¹ or less) to >10,000 L hr⁻¹ (Figure 3).

Every crystalline bedrock lithology can therefore effectively be regarded as an aquifer, capable of supporting water wells yielding economically viable quantities of water. One can speculate that the reason that most lithologies exhibit remarkably similar yield distributions (Figure 3, Krásný and Sharp 2007) is that they have similar, silicate-based matrices with similar geomechanical properties. They may also have been subject to broadly similar recent tectonic stress fields and post-glacial stress-release histories (which are likely to be far more important for hydraulic properties than ancient geological stress histories) (Zoback 1992, Banks et al. 1994). Thus, from a regional point of view, they contain a population of fractures with similar distributions of size and aperture.

One may further speculate that the small differences in median yield between lithologies reflect differences in weathering and geomechanical properties; typically somewhat higher median yields and hydraulic conductivities are found in felsic rocks (e.g., Bäckblom et al. 1997, Olofsson et al. 2001, Knutsson
such as quartzites and granites, which are more brittle and might support larger-aperture, non-self-healing fractures. Lower median yields are typically reported from mafic rocks (e.g., gabbros), anorthosites and shales, which are regarded as being more readily weathered and more ductile and can accommodate strain by larger numbers of smaller or self-healing fractures. The term 'self-healing' refers to fractures in rocks exhibiting ductile or plastic deformation, where fracture apertures can be irreversibly reduced under appropriate ambient stress fields by movement of e.g., clay platelets.

Regardless of the possible explanations for the modest degree of lithological dependence of yield, Morland (1997) found that the median yield of 12,757 Norwegian crystalline bedrock wells was 600 L hr\(^{-1}\) (± 17 L hr\(^{-1}\) at a 95% confidence level), with 25%- and 75%-ile values of 300 L hr\(^{-1}\) and 1500 L hr\(^{-1}\) 2008), such as quartzites and granites, which are more brittle and might support larger-aperture, non-self-healing fractures. Lower median yields are typically reported from mafic rocks (e.g., gabbros), anorthosites and shales, which are regarded as being more readily weathered and more ductile and can accommodate strain by larger numbers of smaller or self-healing fractures. The term 'self-healing' refers to fractures in rocks exhibiting ductile or plastic deformation, where fracture apertures can be irreversibly reduced under appropriate ambient stress fields by movement of e.g., clay platelets.

The high median yields of lithologies such as the Permian sedimentary rocks and lavas (Table 1, Figure 3) can be explained by the fact that groundwater flow occurs, at least partially, through primary features—intergranular pore spaces, chilled/cracked margins or vesicles.

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Regional similarities in the distributions of well yield from crystalline rocks in Fennoscandia (Morland pers. comm.). The median depth of the wells was 52 m. Since Morland’s (1997) analysis, NGU has expended considerable effort on acquiring high-quality data from drillers: the current database (as of March 2008) contains 26,811 records of bedrock well yields, with a median yield of 500 L hr⁻¹ and lower and upper quartile yields of 200 and 1500 L hr⁻¹ (Figure 2). It is suspected that the differences between the 1997 and 2008 analyses may reflect the ways in which ‘zero’ entries in the database have been handled.

Fagerlind (1986) and Gustafson (2002) document the findings from a statistical analysis of the Swedish data set, performed in around 1986. They, too, found that the median yield was 600 L hr⁻¹ from a data set of 59,000 wells. The 25% and 75% percentiles are reported as 270 and 1800 L hr⁻¹, respectively (Figures 2 and 4), and the median depth of the wells was 64 m. For wells drilled in late 1987 and 1988 (N=6630), the median depth and yield were slightly higher at 75 m and 700 L hr⁻¹ (Fagerlind 1989). Some systematic differences in median yield according to lithology can be identified (Struckmeier 1993, Olofsson et al. 2001, Knutsson 2008) and there are some Swedish regions (e.g., Skåne or Scania, as it is sometimes known in English, near the Tornqvist Zone) where Precambrian basement rocks appear to exhibit higher well yields than elsewhere (Table 2, Figure 1).

In Finland, current yield data are not available from any national database. However, examination of a regional database (which is regularly updated) of crystalline rock wells, drilled between 1947 and 2007 in Central Finland, reveals a median yield of 700 L hr⁻¹ (N=1297) with 25%- and 75%-ile yields at 300 and 2000 L hr⁻¹ (Figure 2). The median depth for the boreholes in the data set is 73 m (N=1996) with 25%- and 75%-ile depths of 46 and 109 m (Mäkelä 1994 and recent output from updated database). The mean yield is 1592 L hr⁻¹, demonstrating...
Table 2. Median hydraulic conductivities (m s⁻¹) of drilled wells in different counties of Sweden.

<table>
<thead>
<tr>
<th>County (län)</th>
<th>N</th>
<th>Median hydraulic conductivity (m s⁻¹)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jönköping</td>
<td>3,500</td>
<td>5.0 x 10⁻⁸</td>
<td>Gierup et al. (1999a)</td>
</tr>
<tr>
<td>Västra Götaland</td>
<td>21,000</td>
<td>4.2 x 10⁻⁸</td>
<td>Antal et al. (1999a)</td>
</tr>
<tr>
<td>Västmanland</td>
<td>4,200</td>
<td>5.7 x 10⁻⁸</td>
<td>Bergman et al. (1999b)</td>
</tr>
<tr>
<td>Dalarna</td>
<td>5,000</td>
<td>5.5 x 10⁻⁸</td>
<td>Gierup et al. (1999c)</td>
</tr>
<tr>
<td>Skåne</td>
<td>4,000</td>
<td>2.74 x 10⁻⁷</td>
<td>Gierup et al. (1999d)</td>
</tr>
<tr>
<td>Jämtland</td>
<td>1,000</td>
<td>9.0 x 10⁻⁶</td>
<td>Antal et al. (1999c)</td>
</tr>
<tr>
<td>Örebro</td>
<td>4,900</td>
<td>4.3 x 10⁻⁸</td>
<td>Bergman et al. (1999a)</td>
</tr>
<tr>
<td>Värmland</td>
<td>7,000</td>
<td>5.6 x 10⁻⁶</td>
<td>Fredén et al. (1999)</td>
</tr>
<tr>
<td>Kronoberg</td>
<td>1,800</td>
<td>9.3 x 10⁻⁸</td>
<td>Gierup et al. (1999b)</td>
</tr>
<tr>
<td>Halland</td>
<td>4,000</td>
<td>5.4 x 10⁻⁸</td>
<td>Antal et al. (1999b)</td>
</tr>
<tr>
<td>Norrbotten</td>
<td>2,700</td>
<td>5.0 x 10⁻⁸</td>
<td>Bergman et al. (1998)</td>
</tr>
<tr>
<td>Västernorrland</td>
<td>3,400</td>
<td>3.9 x 10⁻⁸</td>
<td>Antal et al. (1998f)</td>
</tr>
<tr>
<td>Uppsala</td>
<td>7,000</td>
<td>5.9 x 10⁻⁸</td>
<td>Antal et al. (1998a)</td>
</tr>
<tr>
<td>Stockholm</td>
<td>15,000</td>
<td>2.2 x 10⁻⁶</td>
<td>Antal et al. (1998e)</td>
</tr>
<tr>
<td>Södermanland</td>
<td>4,200</td>
<td>2.2 x 10⁻⁸</td>
<td>Antal et al. (1998d)</td>
</tr>
<tr>
<td>Kalmar</td>
<td>2,500</td>
<td>5.9 x 10⁻⁸</td>
<td>Antal et al. (1998b)</td>
</tr>
<tr>
<td>Blekinge</td>
<td>2,100</td>
<td>8.3 x 10⁻⁸</td>
<td>Antal et al. (1998c)</td>
</tr>
</tbody>
</table>

Sum 93,300
Arithmetic mean 6.75 x 10⁻⁸
Weighted mean 5.54 x 10⁻⁸

*In some counties with later (e.g., Mesozoic and Caledonian) cover rocks, only wells drilled in pre-Caledonian basement (turbine) are considered.

Table 3. Statistics for drilled wells in Central Finland, supplied by the database of the Central Finland Regional Environment Centre (now the Central Finland Centre for Economic Development, Transport and the Environment).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Depth Unit</th>
<th>Yield Subset 1947-2007</th>
<th>Yield Granite</th>
<th>Yield Granodiorite</th>
<th>Yield Other intrusive rocks</th>
<th>Yield Subvolcanic / volcanic rocks</th>
<th>Yield Mica gneiss and schist</th>
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</thead>
<tbody>
<tr>
<td>Subset</td>
<td>All wells</td>
<td>All wells Granodiorite</td>
<td>All wells Subvolcanic / volcanic rocks</td>
<td>All wells Mica gneiss and schist</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0 (minimum)</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>23</td>
<td>40</td>
<td>50</td>
<td>40</td>
<td>15</td>
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<td>10</td>
<td>31</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>70</td>
<td>83</td>
<td>150</td>
</tr>
<tr>
<td>25 (1st quartile)</td>
<td>46</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>600</td>
<td>650</td>
<td>700</td>
</tr>
<tr>
<td>50 (median)</td>
<td>73</td>
<td>700</td>
<td>700</td>
<td>600</td>
<td>600</td>
<td>800</td>
<td>500</td>
</tr>
<tr>
<td>75 (3rd quartile)</td>
<td>109</td>
<td>2000</td>
<td>2000</td>
<td>1800</td>
<td>4800</td>
<td>5000</td>
<td>4200</td>
</tr>
<tr>
<td>90</td>
<td>151</td>
<td>4600</td>
<td>4000</td>
<td>4800</td>
<td>5000</td>
<td>5000</td>
<td>4200</td>
</tr>
<tr>
<td>95</td>
<td>181</td>
<td>6000</td>
<td>6000</td>
<td>6000</td>
<td>8000</td>
<td>8000</td>
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</tr>
<tr>
<td>99</td>
<td>247</td>
<td>10000</td>
<td>10000</td>
<td>9000</td>
<td>20000</td>
<td>20000</td>
<td>10000</td>
</tr>
<tr>
<td>100 (maximum)</td>
<td>505</td>
<td>24000</td>
<td>24000</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
<td>10000</td>
</tr>
<tr>
<td>N</td>
<td>1996</td>
<td>1297</td>
<td>393</td>
<td>562</td>
<td>144</td>
<td>85</td>
<td>101</td>
</tr>
<tr>
<td>Mean</td>
<td>84</td>
<td>1592</td>
<td>1574</td>
<td>1531</td>
<td>1869</td>
<td>1498</td>
<td>1768</td>
</tr>
<tr>
<td>Median</td>
<td>73</td>
<td>700</td>
<td>700</td>
<td>600</td>
<td>650</td>
<td>700</td>
<td>864</td>
</tr>
<tr>
<td>Mode</td>
<td>40</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>500</td>
<td>600</td>
<td>600</td>
</tr>
</tbody>
</table>
the skewed character of the data set towards low yields. There is relatively little variation with lithology (Table 3).

Rönkä’s (1983) data for 368 Finnish drilled wells resulted in an arithmetic mean yield of a similar magnitude to the Central Finnish data set—1188 L hr⁻¹—with relatively little lithological dependence. The average depth of wells in his data set was 60 m (N=385). The data of Laakso (1960) consisted of cable-tool wells drilled by a single firm, mainly in the 1950s, with an average depth of 68 m and an arithmetic mean yield of 2460 L hr⁻¹ (N=1108). Hyyppä (1984) considered (for water quality tool wells drilled by a single firm, mainly in the 1950s, with an average dependence. The average depth of wells in his data set was 60 m (N=385). The data of Laakso (1960) consisted of cable-tool wells drilled by a single firm, mainly in the 1950s, with an average depth of 68 m and an arithmetic mean yield of 2460 L hr⁻¹ (N=1108). Hyyppä (1984) considered (for water quality tool wells drilled by a single firm, mainly in the 1950s, with an average)

All of these data sets for Norway and Sweden have not been available for rigorous statistical testing within this study, but examination of Figure 4 reveals approximately straight lines on probability-logarithmic plots, indicating that the distributions are close to log-normal, at least over their central portions. The Central Finnish data set has been log-transformed and tested for nor-

Log-normal distributions

It is widely accepted that the distributions of physical dimensions of fractures (aperture, length) within a population (a fracture set) are typically approximately log-normally distributed (Banks et al. 1996). Power-law distributions are also sometimes considered more appropriate (de Dreuzy et al. 2002) for the distribution of fracture dimensions but, as Mitzenmacher (2004) demonstrates, power-law and log-normal distributions are often very difficult to distinguish, especially at the upper end (at the lower end, differences may be significant, however).

The origin of log-normal distributions in nature is often due to multiplicative or branching processes (Limpert et al. 2001, Mitzenmacher 2004). Bershadskii (2000) has demonstrated that the branching and fragmentation of fractures during their propagation can result in just such a distribution.

If we accept that fracture apertures are approximately log-normally or power-law distributed, it is then a short step to accepting that fracture transmissivities are similarly distributed, given that the transmissivity \( T \) of an ideal (smooth, plane-parallel) fracture is given by the equation (Snow 1969, Walsh 1981):

\[
T = \frac{pgb^3}{12\mu}
\]  

(1)

where \( \rho \) is the density of water, \( g \) is the acceleration due to gravity, \( b \) is the fracture aperture and \( \mu \) is the dynamic viscosity of water. Given a Poisson or negative binomial distribution of fracture frequency and a power-law or log-normal distribution of fracture transmissivity, we should not be wholly surprised to find an approximately log-normal distribution of yields in databases of boreholes drilled in hard rock aquifers (e.g., Moore et al. 2002, Figure 4). This is understandable if we consider the observations that:

- the yields documented in such databases are typically short-term yields and are probably controlled by the transmissivity of the ‘feeder’ fractures in the immediate vicinity of the borehole, rather than the properties of the broader aquifer network storage (including connectivity) or the available recharge. It is these fractures which connect the well to a wider network of fracture storage and are usually the limiting factor for a crystalline bedrock well’s yield in the short term and in the case of relatively poorly yielding wells (Banks 1992).
- the yield of wells in crystalline bedrock aquifers is typically dominated by groundwater inflow from a very small number (or sometimes just one) of fractures (e.g., Carlsson and Olsson 1977), which itself is a consequence of a fracture transmissivity distribution skewed towards low yields. Such an observation argues against the utility of the concept of hydraulic conductivity (an intrinsic property that presupposes a continuum approach) when describing properties derived from single boreholes, while the extrinsic (fracture-specific) concept of transmissivity is more tenable.

In this study, we have regarded transmissivity as the primary hydraulic property that can be estimated from consideration of well yields. In most simple hydraulic analysis methods (e.g., Theis 1935, Cooper and Jacob 1946, Logan 1964), transmissivity is proportionately related to specific capacity (the ratio of yield to drawdown) and it is specific capacity that can be inferred from the well yield data considered in this paper. Average hydraulic conductivity can be regarded as a secondary parameter calculated as transmissivity divided by the thickness of the tested interval.

Moore et al. (2002) regard the distribution of yields in their New Hampshire data set as log-normally distributed, while Banks (1998) has used Norwegian data to demonstrate that the yields of wells from individual crystalline rock lithologies are indistinguishable from a log-normal distribution at a 95% confidence level. Previous studies have indicated yield data from Central Finland to be approximately log-normally distributed (Mäkelä 1994). Morland’s (1997) and Fagerlind’s (1986) full raw data sets for Norway and Sweden have not been available for rigorous statistical testing within this study, but examination of Figure 4 reveals approximately straight lines on probability-logarithmic plots, indicating that the distributions are close to log-normal, at least over their central portions. The Central Finnish data set has been log-transformed and tested for nor-
mality using the Shapiro-Wilk and Kolmogorov-Smirnov tests. The former test yielded a W-value of 0.916 and the latter a D-value of 0.103, respectively (N=1300), which were not adequate to support a hypothesis of log-normality at a 95% confidence level. These relatively rigorous statistical tests require a good fit across the entire distribution to be satisfied, while, in the Finnish data set, the deviations from log-normality occurred at the extremes of the distribution, with the main, central portion of the data closely resembling log-normality (Figure 5a).

As with fracture dimensions (see above), there is some debate as to whether a power-law distribution is more appropriate for well yields and hydraulic properties of crystalline rocks. Gustafson and Fransson (2006) argue for a power law distribution of fracture transmissivities. The Central Finnish data set has also been Box-Cox transformed, using a power law algorithm, and tested for normality using the same tests, yielding a W-value of 0.991 and a D-value of 0.0611 (Figure 5b). Although these values were still not wholly adequate to demonstrate a normalising transformation, the power-law transformation is qualitatively far more convincing than the logarithmic transformation, with a much better fit to the tails of the distribution.

An heuristic approach to median transmissivity and hydraulic conductivity

It has been very common in nations dominated by poorly weathered crystalline rock aquifers to use well yield as a proxy for aquifer transmissivity (Jetel and Krásný 1968, Krásný 1975, Carlsson and Carlstedt 1976, Wladis and Gustafson 1999). Furthermore, several geotechnical methods for determining the hydraulic conductivity of intervals of hard rock borehole essentially measure the flow of water between borehole and aquifer for a given pressure differential. The authors have thus undertaken an heuristic approach (i.e., an analysis undertaken in the spirit of scientific ‘playfulness’ and discovery, on the explicit understanding that some of the underlying assumptions and conceptual models are grossly oversimplified) to ascertain whether the observed distributions of well yield can be explained by a characteristic distribution of transmissivity.

Banks (1992) examined a large variety of published relationships between crystalline rock well yield and apparent transmissivity $T_a$ and found that most were of the form:

$$T_a = \frac{Q}{\alpha s}$$  \hspace{1cm} (2)

where $Q/\alpha$ is the short-term specific capacity (ratio of yield $Q$ to drawdown $\alpha$) and $\alpha$ is a constant. Banks (1992) concluded with recommending a value of $\alpha=0.9$, although Wladis and Gustafson (1999) suggest a somewhat lower value of $\alpha=0.45$ from Swedish studies. For the purposes of this paper, we will assume a value of $\alpha=0.7 \pm 0.2$ (the error margins cited in this paper should be regarded as estimates of limits of confidence; the value cited here is selected to span the majority of $\alpha$ values suggested in the above studies).

If we know the yield of a well in crystalline bedrock, we can thus make an estimate of the apparent aquifer transmissivity, provided we can estimate the drawdown in the well corresponding to that yield. Let us assume that the depth to rest water level
in a crystalline bedrock borehole is \( d \) and the depth of the well is \( D \). In low-yielding crystalline bedrock wells in Scandinavia, it is common practice to measure yield by one of the following methods:

1) the amount of water blown out of the well over a modest period of time by compressed air introduced at the base of the well on completion of drilling (in which case the ‘drawdown’ can be taken to be the depth of the well minus the rest water level; i.e., \( \Delta D = D - d \)).

2) a recovery test, where the hole is emptied by a pump near the base of the well, the well is ‘skim pumped’ at that level for a short period and thereafter the inflow (\( Q \)) is determined as:

\[
Q = \pi r_w^2 \Delta h \Delta t
\]

by measuring the recovery of the water level \( \Delta h \) over a given time interval \( \Delta t \), where \( r_w \) is the radius of the drilled well. In this case, the appropriate measure of drawdown to associate with the yield is probably around \( 2/3(D-d) \) (i.e., we are assuming that the interval over which recovery is measured is \( 2/3 \) down the borehole, relative to rest water level).

3) a pumping test, where a submersible pump cuts in and out in response to water level recovery in the borehole following a short period of pumping. Again, it is suggested that an appropriate measure of drawdown is probably around \( 1/2(D-d) \) to \( 2/3(D-d) \).

The vast majority of wells in the Scandinavian bedrock well databases are drilled boreholes. The depth of these crystalline rock boreholes is commonly between 40 and 100 m, with depths of around 70 m being typical. The rest water level is typically rather near the surface (0–15 m), as a water table which closely follows the topography is a characteristic of low permeability crystalline rock terrain (Banks and Robins 2002). A typical value of \( (D-d) \) can thus be estimated as \( 60 \pm 10 \) m. A typical value of drawdown is thus \( 2/3(D-d) = 40 \pm 7 \) m.

Clearly, the value of \( T_r \) has rather little meaning for a single well, but if we consider a large enough data set, we are in a position to estimate the median apparent transmissivity of crystalline hard rock aquifers in Fennoscandia. We may assume:

1) the median well yield in such aquifers is \( 650 \pm 50 \text{ L hr}^{-1} = 15.6 \pm 1.2 \text{ m}^2 \text{ d}^{-1} \)

2) the corresponding drawdown is estimated as \( 40 \pm 7 \) m.

3) the constant of proportionality \( \alpha = 0.7 \pm 0.2 \)

Equation (2) thus provides us with a median value of transmissivity of \( T_r = 0.56 \pm 0.30 \text{ m}^2 \text{ d}^{-1} = 6.4 \times 10^{4} \pm 3.4 \times 10^{4} \text{ m}^2 \text{ s}^{-1} \).

Statistically, this method is flawed, as it assumes that the expected value of specific capacity is simply the ratio between the expected value of the yield and that of drawdown (which is related to depth). It thus effectively assumes that there is no correlation between yield and depth. In fact, there is known to be a dependence of yield on depth due to at least two factors:

(a) the observation that borehole depths are generally shallower in ‘better’ crystalline rock aquifers, as the driller often stops drilling once adequate water has been achieved (Rönkkö 1993, Morland 1997), and (b) the total effective transmissivity of the aquifer increases with drilled depth, although the transmissivity of individual fractures tends to decrease with drilled depth in a nonlinear manner (see below). As these relationships are not necessarily linear and are different in nature (the first being a ‘drilling-psychological’ factor and the second geotectonic in nature), any correlation between depth and yield has been ignored in the above estimate.

**Bulk transmissivity**

For networks of fractures, whose dimensions are log-normally or power-law distributed, many researchers have argued that, at scales in excess of a given threshold (the ‘Representative Elementary Volume (REV)’), the bulk transmissivity of the network (i.e., its overall ability to transmit water in response to head gradients) is best described by the geometric mean of the transmissivities (or hydraulic conductivities) of the individual fracture elements (e.g., de Dreuzy et al. 2002, Hunt 2005). The REV can be regarded as the volume/dimension at which there are only small changes in bulk hydraulic conductivity for small changes in sample size or sample location. It is a concept that is not limited to fractured rock aquifers, but which can also be applied to other forms of aquifer heterogeneity. For 2D isotropic systems, Renard and de Marsily (1997) and Renard et al. (2000) regard the geometric mean of permeabilities as being a good solution to bulk permeability and for 3D systems as being a plausible, though not perfect, estimate.

Moreover, for a distribution of fracture transmissivities or hydraulic conductivities that is approximately log-normal, the median value of that distribution will closely approximate the geometric mean (an observation empirically confirmed by researchers such as Broch and Kjøholt 1994). We can thus conclude that our estimated median value of transmissivity of 0.56 m² d⁻¹ essentially represents the bulk transmissivity of the upper 70–80 m of crystalline bedrock that are penetrated by typical Scandinavian hard rock wells.

We should, of course, bear in mind that the transmissivity represented by these relatively shallow bedrock wells will only be a proportion of the total transmissivity of the rock ‘column’. Swedish work suggests that crystalline bedrock hydraulic conductivity (measured as \( K \) : the transmissivity divided by the tested interval of a borehole) decreases strongly with depth according to the relationship of the type shown in Figure 6 (which is derived specifically for a site as Oskarshamn in Sweden by SKB 2006). If such a relationship is more universally applicable, it would imply that the bulk transmissivity of the portion of aquifer down to 100 m may only account for 45% of the total transmissivity of the rock mass. Moreover, the upper 70
m and 50 m represent only 35% and 26% of the total transmissivity, respectively. Here, the ‘total transmissivity’ should be understood as the sum of the fracture transmissivity down to an (unspecified) depth where fractures are effectively hydraulically ‘closed’ by the ambient stress field.

It is tempting to endeavour to convert our estimate of bulk apparent transmissivity to a value of bulk hydraulic conductivity $K_b$. We should here note that, at small scales, hydraulic conductivity is usually regarded as scale-dependent (Brace 1984, Clauser 1992). Only above a given REV (which Gustafson, 1986, estimates as being at least 1000 m$^3$) do we find that we can begin to consider a consistent value of bulk hydraulic conductivity, which can be considered ‘equivalent’ to the hydraulic conductivity of an analogous porous medium and which is not highly dependent on scale or location (Renard and de Marsily 1997). If the typical saturated depth (i.e., the section of rock hydraulically ‘tested’) of a Scandinavian water well borehole is around 60 m, it is possible to say that an estimated geometric mean transmissivity of $0.56 \pm 0.30$ m$^2$ d$^{-1}$ for $d=70$ m.

Comparison with other international data sets

It is hoped that this paper will stimulate discussion and encourage other nations to collate and publish statistical summaries of yield data from water wells in hard rock aquifers. We hope that this will allow us to ascertain if the consistency of yield distributions in wells from Fennoscandia is reproduced from other tectonic settings and other climates, or whether well yield distributions are significantly influenced by other factors, such as recent weathering environment (Banks et al. 1998). Some data are already published from other crystalline bedrock regions of the world (Table 4), which offer a tantalising suggestion that the ‘median yield=600–700 L hr$^{-1}$’ rule may have broader application than just Fennoscandia, as Krásný and Sharp (2007) have suggested. It was noted, in the Introduction, that the conclusions of this paper might only be expected to be valid in recently glaciated crystalline bedrock terrain—i.e., in true fractured aquifers, where a porous, permeable, weathered regolith aquifer is absent. Nevertheless, some of the case studies in Table 4 seem to suggest that, even where a weathered regolith could be expected, some of the observations regarding well yield distribution seem (surprisingly) to be valid.

Conclusions

The two main (and admittedly provocative) conclusions of this paper are:

1) Wells drilled in crystalline bedrock in recently glaciated, relatively unweathered terrain in Fennoscandia exhibit a wide range of yields. The distribution of these yields exhibits, however, a consistent median value of 600–700 L hr$^{-1}$. Analysis of subsets of data according to factors such as lithology, topography, proximity to lineaments etc., often reveals a relatively weak correlation with such factors. The ability to deliver realistic and helpful prognoses when prospecting for groundwater in such terrain thus requires an understanding of statistics every bit as much as skills in identifying deter-
Table 4. Hydraulic properties of crystalline rock terrains cited in worldwide literature and compared with Fennoscandian values (this paper).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Location</th>
<th>Result</th>
<th>Reference</th>
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<tbody>
<tr>
<td><strong>Northern terrains</strong></td>
<td></td>
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<td></td>
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<tr>
<td>Median well yield 600–700 L hr⁻¹</td>
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<tr>
<td>Northern terrains</td>
<td>3000 wells, British Columbia, Canada</td>
<td>Median well yields from 340 to 1150 L hr⁻¹ in different lithological groups (see Figure 3). Median of 700–900 L hr⁻¹ typical.</td>
<td>Kohut (2006)</td>
</tr>
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<td></td>
<td>196 boreholes in igneous and metamorphic rocks, Manitoba, Canada</td>
<td>Median yield = 700 L hr⁻¹</td>
<td>Betcher et al. (1995)</td>
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<td></td>
<td>433 boreholes in metamorphic bedrock (excluding limestone), southern Vancouver Island, Canada</td>
<td>Median yield = 900 L hr⁻¹</td>
<td>Kenny et al. (2006)</td>
</tr>
<tr>
<td></td>
<td>Caledonian (Appalachian) terrain of Maine, USA</td>
<td>Median yield = 3 to 6 US gallons/minute (680 to 1360 L hr⁻¹) for different map sheets</td>
<td>Maine Geological Survey (2007)</td>
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<td></td>
<td>Domestic bedrock wells, Maine, USA</td>
<td>Median yield = 4 US gallons min⁻¹ = c. 900 L hr⁻¹</td>
<td>Maine State Planning Office (2001)</td>
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<td></td>
<td>20,308 drilled wells in New Hampshire, USA</td>
<td>Median yield = 6 US gallons/min⁻¹ = 1360 L hr⁻¹</td>
<td>Moore et al. (2002)</td>
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<td><strong>Tropical terrains</strong></td>
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<td></td>
<td>Crystalline basement, eastern Chad</td>
<td>Overall well ‘success rate’ of 30–40% and a median yield of 1500–2000 L hr⁻¹ in ‘successful’ wells</td>
<td>Well database held in N’Djamena. Cited in Misstear et al. (2006)</td>
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<td></td>
<td>Masingto, Zimbabwe</td>
<td>30–40% of wells have yields less than 360 L hr⁻¹</td>
<td>Wright (1989), Herbert et al. (1993)</td>
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<tr>
<td></td>
<td>Drilled wells, northern Niger</td>
<td>Median yield varies from 430 to 2010 L hr⁻¹ in different lithological groups (see Figure 3).</td>
<td>Barratt and Puyoo (1984)</td>
</tr>
<tr>
<td></td>
<td>536 wells in Precambrian gneisses and schists, Kandy region, Sri Lanka (median depth = 76 m)</td>
<td>Median yield = 780 L hr⁻¹</td>
<td>Johansson (2005)</td>
</tr>
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<td></td>
<td>380 wells in Precambrian basement, NE Brazil</td>
<td>Median yield = 1800 L hr⁻¹</td>
<td>Rebouças (1999)</td>
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<tr>
<td><strong>Bulk transmissivity 0.56 ± 0.30 m² d⁻¹</strong></td>
<td>22 drilled wells, Uganda (regolith excluded by casing)</td>
<td>Median transmissivity = 0.8 m² d⁻¹</td>
<td>Howard et al. (1992)</td>
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<td><strong>Northern terrains</strong></td>
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<td></td>
<td>To depths of 1 km, Europe and Canada</td>
<td>Typical bulk conductivity = 10⁻⁶ to 10⁻⁸ m s⁻¹</td>
<td>Stober and Bucher (2007)</td>
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<td></td>
<td>c. 140,000 wells in Norway and Sweden</td>
<td>‘Average’ conductivity = 10⁻⁶ to 10⁻⁸ m s⁻¹</td>
<td>Henrikse (2008) Figure 7</td>
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<td></td>
<td>Upper few hundred metres of Canadian shield, Canada</td>
<td>10⁻⁶ to 10⁻⁸ m s⁻¹</td>
<td>Dickin et al. (1984).</td>
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<td></td>
<td>To 100 m depth, Altnabreac, Scotland</td>
<td>Range = 10⁻⁶ to 10⁻⁸ m s⁻¹</td>
<td>Mather and Sargent (1986)</td>
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<td><strong>Central European terrains</strong></td>
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<td></td>
<td>Harz Mountains, Germany</td>
<td>Regional conductivity = 3.0 x 10⁻⁷ m s⁻¹</td>
<td>Maloszewski et al. (1999)</td>
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<tr>
<td></td>
<td>Drainage study at 170 m depth, Limoges, France</td>
<td>‘Average’ conductivity = 10⁻⁷ to 10⁻⁸ m s⁻¹</td>
<td>Mather and Sargent (1986)</td>
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</table>
ministic geological factors (Banks et al. 2005).

2) The approximately log-normal distribution of well yields in crystalline bedrock terrain, coupled with a median yield of some 600—700 L hr⁻¹, implies that the bulk transmissivity of the upper 70–80 m or so of Fennoscandian crystalline bedrock crust is around 0.56 ± 0.30 m² d⁻¹, although this may only represent around one third of the total transmissivity of the full depth of the rock mass. Considering a typical saturated depth of a borehole, leads us to estimate the bulk hydraulic conductivity of a similar aquifer interval to be around 1 x 10⁻⁷ m s⁻¹. This last estimate must be treated with considerable scepticism, given the strong depth dependence of hydraulic conductivity and the varying depth of wells used in the analysis.

Knowledge of the statistical distribution of hydraulic properties of crystalline bedrock is a prerequisite for a cost-effective drilling strategy in such aquifers. Knowledge of bulk transmissivity will also have relevance to assessing the water balance and available water resources of bedrock aquifer catchments and to the assessment of the significance of advection of heat with groundwater in the underground thermal energy storage (UTES) systems that are becoming increasingly common throughout Scandinavia.

Acknowledgements
The authors wish to thank Ritva Britschgi of the Finnish Environment Institute (Suomen ympäristökeskus) for her interest and enthusiasm during the preparation of this paper. They also thank Nick Robins of the British Geological Survey and Dr. Noelle Odling of Leeds University for their comments and constructive reviews of the manuscript.

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